JOURNAL OF COMPUTATIONAL PHYSICS 6, 152–153 (1970)

Letter to the Editor

A Rational Approximation to an Integral Appearing in Glow Curve Theory

Chen [1] discussed the calculation of

$$F(T, E) = \int_0^T \exp(-E/kT')\, dT' \tag{1}$$

which arises in glow curve theory. His approach was based on an asymptotic series for which he derived a termination criterion and an error estimate.

I propose the simple approximation

$$F(T, E)/T = \exp(-X)(X + 3.0396)/(X^2 + 5.0364X + 4.1916) \tag{2}$$

where $X = E/kT$. This is based on a rational form developed by Hastings [2], but the constants are slightly rounded from those given in [3] for $X \geqslant 10$, which is the range of interest in the present application.

Table I compares values obtained from (2) with those from an exact expression

$$F(T, E)/T = \exp(-X) \cdot [1. - X \exp(-X)\, E_1(X)] \tag{3}$$

using values from [3] for the exponential integral E_1. If greater accuracy is required more terms must be used in (2) and the constants specified to more significant figures. Hastings provides a set which will give 8-place accuracy for $x \geqslant 1$.

TABLE I

Comparison of Approximate and Exact Values of $F(T, E)/T = \frac{1}{T}\int_0^T \exp(-E/kT')\, dT'$

E/kT	Equation (2) (Approximate)	Equation (3) (Exact)
8	0.3143788(−4)	0.3143764(−4)
9	0.1138380(−4)	0.1138362(−4)
10	0.3830317(−5)	0.3830240(−5)
20	0.9405092(−10)	0.9404857(−10)
25	0.5157086(−12)	0.5156945(−12)

REFERENCES

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2. C. HASTINGS, JR., "Approximations for Digital Computers," Princeton Univ. Press, Princeton, N. J., 1955.
3. M. ABRAMOWITZ AND I. A. STEGUN, ED., "Handbook of Mathematical Functions," Chap. 5. National Bureau of Standards, Washington, D. C., 1964.

RECEIVED: November 10, 1969

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